DOUBLE CIRCULAR-TRIANGULAR SIX-DEGREES-OF-FREEDOM PARALLEL ROBOT

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Abstract
This paper describes a new structure of a six-DOF parallel robot. First, a known planar three-DOF double-triangular structure is modified by replacing the stationary triangle with a circle. It increases the work envelope considerably especially when rotational motions are required. The ability for unlimited rotational motion allows extending the structure into six-DOF by using two sets of stationary circles and moveable triangles. Each set can actuate the moving triangle in a planar three-DOF motion and hence actuate a line connecting the centers of the movable triangles in four-DOF. The robot's end-effector is attached to a link along this line while rotation about and translation along this line are obtained by the additional rotational DOF of the movable triangles. The solution of the direct kinematics of this six-DOF manipulator is given in a closed-form and it is shown that at most, four different solutions exist.

1. Introduction

The kinematic structure of most contemporary robots is an open kinematic chain structure (known also as serial manipulators). Only relatively few commercial robots are composed of a closed kinematic chain (parallel) structure. However, the increasing interest in parallel robots points to the potential embedded in this structure which has not yet been fully exploited. The advantages of parallel robots as compared to serial ones are:

higher payload-to-weight ratio since the payload is carried by several links in parallel,
higher accuracy due to non-cumulative joint error,
higher structural rigidity, since the load is usually carried by several links in parallel and in some structures in compression-traction mode only,
location of motors at or close to the base,
simpler solution of the inverse kinematics equations.

Conversely, they suffer from smaller work volume, singular configurations and a more complicated direct kinematic solution (which is usually not required for control purposes).

Examples of different structures of parallel manipulators are given, for example, in [Hunt, 1983; Innocenti and Parenti-Castelli, 1994; Lin et al., 1992; merlet, 1994; Pierrot et al., 1991; Tahmasebi and Tsai, 1995; Ben-Horin and Shoham, 1996; Waldron et al., 1989]. A comprehensive atlas of parallel robots was composed by Merlet and can be found in the web site [http:// www.inria.fr/ prisme/ personnel/ merlet/ merlet_eng.html].

This paper presents a new type of a parallel robot that is a modification of and an extension to six-Degrees-Of-Freedom (DOF) of the three-DOF planar parallel robot presented in [Mohammadi, Zsombor-Murray and Angeles, 1993]. The structure presented in the above mentioned paper is of two triangles, one stationary and one moveable, connected at three points, one at each side, by a combination of revolute and prismatic joints. The prismatic joints are actuated while the revolute joints are passive. This constitutes three-DOF of planar motion - two translational and one rotational - of the movable triangle (see Fig. 1).

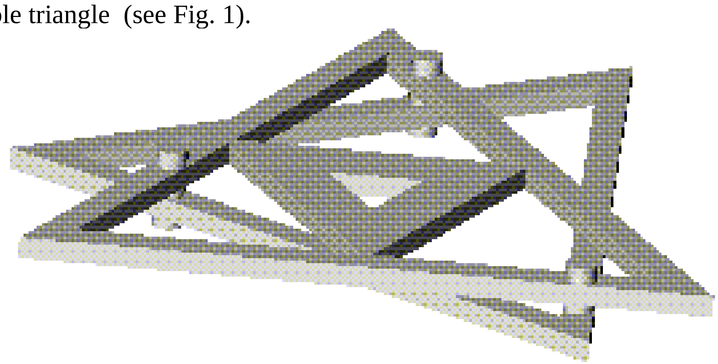

Fig. 1: Double-triangular planar manipulator

Observing the work envelope of this structure, it seems that the area covered by the moveable triangle's center-of-gravity (output link), especially when a combination of translational and rotational motion is required, is relatively small. Fig. 2 depicts the work volume of the double-triangular manipulator for a translational motion and for a combination of translational and rotational motion. If, for example, a rotational motion of 55 degrees is required, then the useful work envelope is too small.

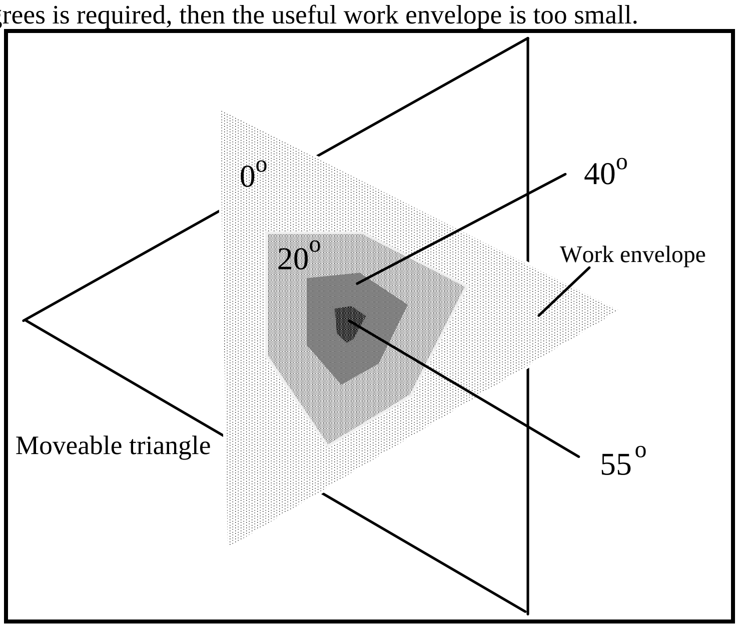

Fig. 2: Work envelope of the Double-triangle manipulator for different rotations

In this paper a modification of this structure is suggested that increases considerably the planar work envelope. The planar manipulator is described in the next section and a combination of two such designs which enable construction a six-DOF parallel robot, is described in section 3.

2. Circular-Triangular Three-DOF Planar Manipulator

In order to increase the work envelope it is suggested to replace the stationary triangle by a circle. This permits a considerable increase of the robot's work envelope especially in rotational motion - theoretically, unlimited rotation about an axis normal to the plane is possible. This structure is realizable by locating the passive prismatic joints, connected to the moveable triangle's sides, along the circumference of the stationary circle (see Fig. 3). In fact, any three points moving along any curve serve this purpose as long as these curves intersect the triangle's sides (not at a right angle).

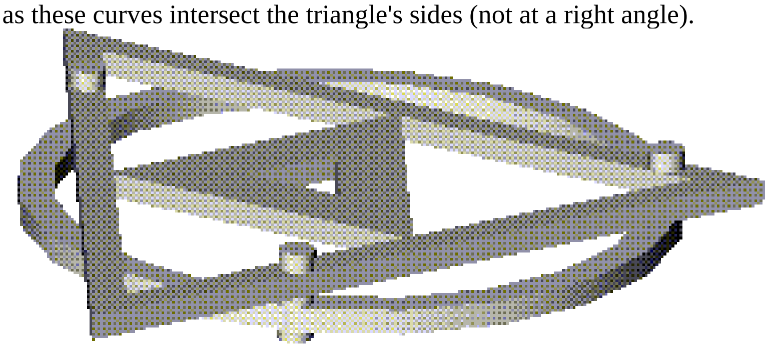

Fig. 3: The Circular-triangular planar manipulator

It is easy to show that the maximal work volume is obtained when the ratio of the circle diameter to the equilateral triangle side is $\sqrt{3/2}$. The work volume of the moveable triangle center-of-gravity with a fixed orientation and with 360^{o} rotation, is shown for this case, in Fig. 4.

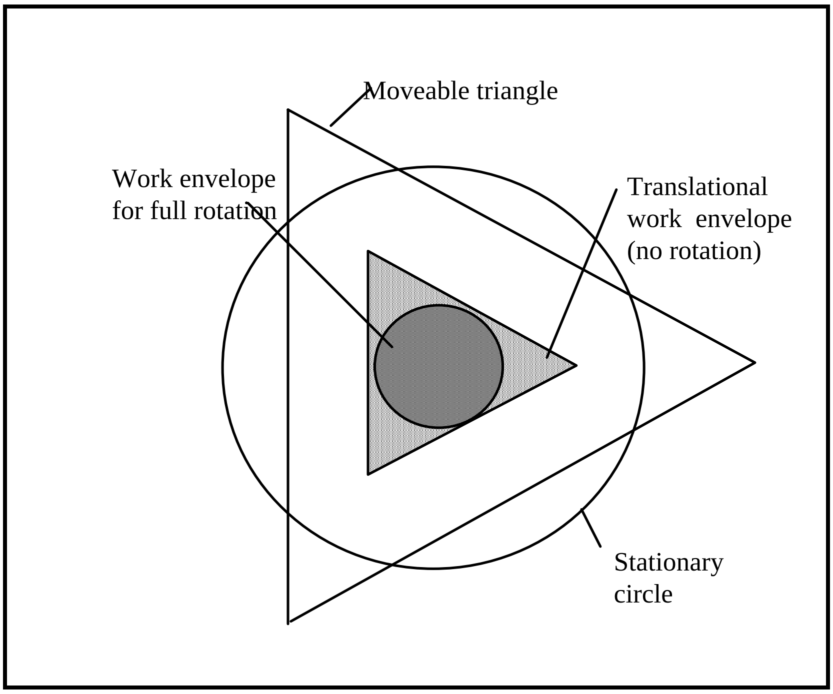

Fig. 4: The work envelope of circular-triangular manipulator

The central area is the "dexterous region" where the triangle can be placed at any required orientation. The ability to have full rotation of the moveable triangle is a key feature, from a practical point of view, in the extension of this design into a six-DOF robot as is shown in the next section.

3. Double Circular-Triangular Six-DOF Parallel Robot

Two sets of the structure described above are used next to construct a six-DOF robot. Each planar structure contributes three-DOF in a plane - moves the triangle's center-of-gravity in the plane and rotates the triangle about an axis perpendicular to this plane. With two such sets, a line connecting the triangles' centers is actuated in four-DOF. The output link is located along this line where the other two-DOFs are obtained by controlling the rotational motion of the moveable triangles. Each triangle's center contains a U joint which is connected to the output link at one triangle's center through a prismatic joint and at the other through a helical joint (nut and a lead screw). Rotational motion of the output link about the line connecting the centers is achieved by rotating the first moveable triangle (the prismatic joint) while motion along the line is achieved by rotating both triangles at different angles. The structure of the six-DOF double circular-triangle parallel robot is shown in Fig. 5.

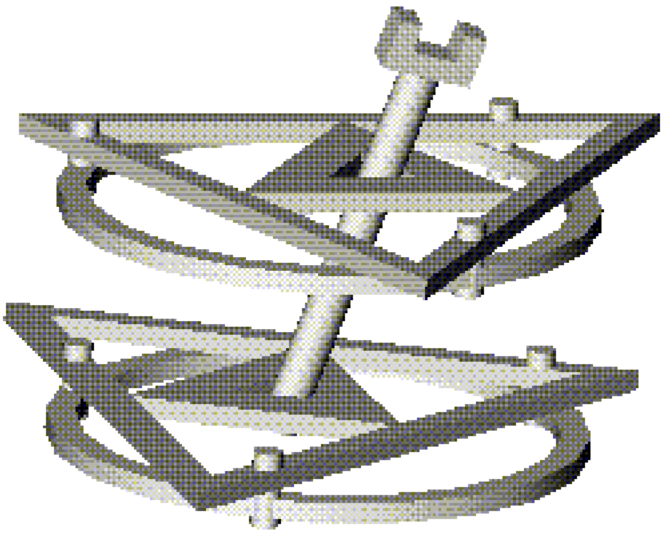

Fig. 5: The Double Circular-Triangular six-DOF robot

4. Direct Kinematics

While the solution of parallel robots inverse kinematics is usually trivial, the direct kinematic is more complicate. In fact, in most cases the solution is reduced to a solution of a high degree polynomial equation. Once the polynom is obtained it means that the kinematics is solved in a closed-form even though the high degree polynom has only a numerical solution. In the present structure a solution of the six-DOF parallel robot is obtained next in closed-form.

4.1 DIRECT KINEMATICS OF THE THREE-DOF PLANAR CIRCULAR-TRIANGLE MANIPULATOR

We start the kinematic analysis by first solving the planar circular-triangular case. We solve the direct kinematics of this structure in a different way than in [Mohammadi, Zsombor-Murray and Angeles, 1993].

The direct kinematic problem is to determine the position of the moveable triangle's center and its orientation from a known location of three points, one at each side of the triangle (in our case these points move along a circle but the exact curve is immaterial for the present solution).

Referring to Fig. 6, the problem is to find the position of point **O**' and angle θ from the known positions of points A, B and C. The lengths L_1, L_2, and L_3 of the triangle's sides are also given.

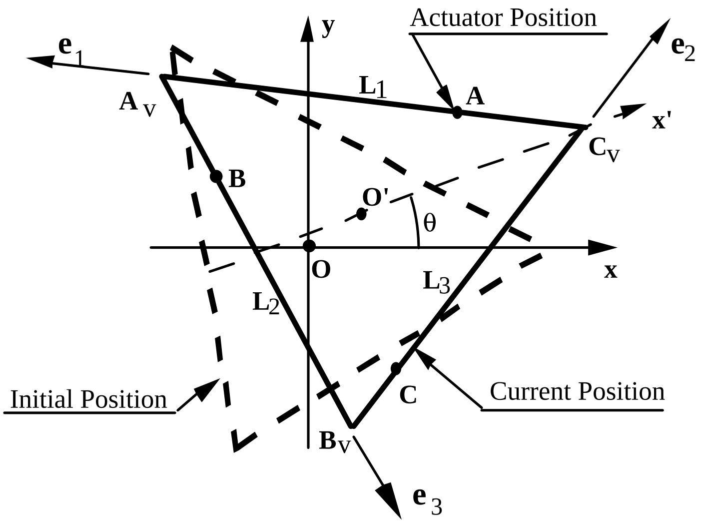

Fig. 6: Moveable Triangle at initial and current positions

We introduce unit vectors, $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$, along each side's direction. Since the triangle is considered a rigid body, the mutual orientation of these vectors remains unchanged. Hence, as the unknown variable we denote the direction cosine of one unit vector, for example, $\mathbf{e}_1$:

$$\mathbf{e}_1 = [a \quad b \quad 0]^T \tag{1}$$

from which the two other unit vectors are obtained:

$$\mathbf{e}_2 = [ac_\alpha - bs_\alpha \quad as_\alpha + bc_\alpha \quad 0]^T, \quad \mathbf{e}_3 = [ac_\beta - bs_\beta \quad as_\beta + bc_\beta \quad 0]^T, \tag{2}$$

where a and b are known and s_i, c_i denote sin i and cos i , respectively.

Next, we use the fact that the moment of each triangle's side about its center-of-gravity is a constant and equal to two thirds of the triangle area, S:

$$L_1|\mathbf{e}_1 \times \mathbf{r}_{OA}| = L_2|\mathbf{e}_2 \times \mathbf{r}_{OB}| = L_3|\mathbf{e}_3 \times \mathbf{r}_{OC}| = \frac{2}{3}S \tag{3}$$

where $\mathbf{r}_{oA}$, $\mathbf{r}_{oB}$, $\mathbf{r}_{oC}$ are vectors from the triangle center to one point on each triangle side.

The following three equations are obtained from Eq. (3):

$$A_x b - A_y a - bx_O + ay_O = \frac{2}{3L_1}S, \tag{4}$$

$$B_x(as_\alpha + bc_\alpha) - B_y(ac_\alpha - bs_\alpha) - (as_\alpha + bc_\alpha)x_O + (ac_\alpha - bs_\alpha)y_O = \frac{2}{3L_2}S$$

$$C_x(as_\beta + bc_\beta) - C_y(ac_\beta - bs_\beta) - (as_\beta + bc_\beta)x_O + (ac_\beta - bs_\beta)y_O = \frac{2}{3L_3}S,$$

where (x_O, y_O) is the current position of the triangle center and (A_x, A_y) (B_x, B_y) (C_x, C_y) are the current positions of point A, B and C respectively.

Multiplying the first equation of (4) by $\left(s_\beta c_\alpha - c_\beta s_\alpha\right)$, the second by $\left(-s_\beta\right)$ and the third by s_α and summing up, one obtains one linear equation in the unknowns *a* and *b*:

$$N_1 a + N_2 b = P\,, \tag{5}$$

where:

$$N_1 = -A_y\left(s_\beta c_\alpha - c_\beta s_\alpha\right) - s_\beta\left(B_x s_\alpha - B_y c_\alpha\right) + s_\alpha\left(C_x s_\beta - C_y c_\beta\right),$$
$$N_2 = A_x\left(s_\beta c_\alpha - c_\beta s_\alpha\right) - s_\beta\left(B_x c_\alpha + B_y s_\alpha\right) + s_\alpha\left(C_x c_\beta + C_y s_\beta\right),$$
$$P = \frac{2S}{3}\left(\frac{s_\beta c_\alpha - c_\beta s_\alpha}{L_1} - \frac{s_\beta}{L_2} + \frac{s_\alpha}{L_3}\right).$$

Noting that $\mathbf{e}_1$ is a unit vector,

$$a^2 + b^2 = 1, \tag{6}$$

then Eq. (5) has the following quadratic solution for *a* and *b*:

$$a = \frac{PN_1 \pm N_2\sqrt{N_1^2 + N_2^2 - P^2}}{N_1^2 + N_2^2}\,, \quad b = \frac{PN_2 \mp N_1\sqrt{N_1^2 + N_2^2 - P^2}}{N_1^2 + N_2^2}\,. \tag{7}$$

Note, that there exists at most, two different solutions for a triangle orientation such that its sides pass through three given points. This result is in accordance with that one obtained in Mohammadi, Zsombor-Murray and Angeles' paper.

The position of the moveable triangle is obtained by multiplying the second and third equations of (4) by $c_\alpha / s_{\alpha-\beta}$ and $-c_\beta / s_{\alpha-\beta}$,respectively, ($s_{\alpha-\beta} = \sin(\alpha - \beta)$ which does not vanish), and summing these equations. This results in the following two linear equations in (x_O, y_O) for triangle's center position:

$$\begin{aligned} -bx_O + ay_O &= T_1 \\ ax_O + by_O &= T_2 \end{aligned} \tag{8}$$

where:

$$T_1 = \frac{2}{3L_1}S - A_x b + A_y a\,,$$
$$\begin{aligned} T_2 = {} & \frac{c_\alpha}{s_{\alpha-\beta}}\left[\frac{2}{3L_2}S - B_x\left(as_\alpha + bc_\alpha\right) + B_y\left(ac_\alpha - bs_\alpha\right)\right] \\ & - \frac{c_\beta}{s_{\alpha-\beta}}\left[\frac{2}{3L_3}S - C_x\left(as_\beta + bc_\beta\right) - C_y\left(ac_\beta - bs_\beta\right)\right] \end{aligned} \tag{9}$$

The determinant of this system of linear equations is equal to -1, hence its solution is:

$$x_O = -bT_1 + aT_2\,,$$

$$y_O = aT_1 + bT_2 . \tag{10}$$

If vertices location are also required, then the following equations are used:

$$\mathbf{r}_{\mathrm{A_v}} = \mathbf{r}_{\mathrm{A}} + \frac{\left|\mathbf{r}_{\mathrm{AB}} \times \mathbf{e}_2\right|}{\left|\mathbf{e}_1 \times \mathbf{e}_2\right|}\mathbf{e}_1, \qquad \mathbf{r}_{\mathrm{B_v}} = \mathbf{r}_{\mathrm{B}} + \frac{\left|\mathbf{r}_{\mathrm{BC}} \times \mathbf{e}_3\right|}{\left|\mathbf{e}_2 \times \mathbf{e}_3\right|}\mathbf{e}_2,$$
$$\mathbf{r}_{\mathrm{C_v}} = \mathbf{r}_{\mathrm{C}} + \frac{\left|\mathbf{r}_{\mathrm{CA}} \times \mathbf{e}_1\right|}{\left|\mathbf{e}_3 \times \mathbf{e}_1\right|}\mathbf{e}_3, \tag{11}$$

where $\mathbf{r}_{A_v}$, $\mathbf{r}_{C_v}$, $\mathbf{r}_{B_v}$ are vectors from the triangle center to vertices A, B and C, respectively.

Eqs. (8) and (10) are the closed-form forward kinematics solution of the planar circular-triangle manipulator.

4.2 DIRECT KINEMATICS OF THE SIX-DOF DOUBLE CIRCULAR-TRIANGLE MANIPULATOR

The six-DOF double circular-triangular manipulator consists of two circular-triangular manipulators positioned, in the next example, parallel to each other. The above algorithm is therefore, used to solve each triangle center's location, and thus the line passing through both centers is determined. For a given six points, each on one triangles sides, there are at most, four different solutions for this line and, in our case, for the forward kinematics.

The solution for the extension of the end-effector along the output link and its rotation, depends on the specific structure of the manipulator (axes of the U joints, location of the lead screw's nut and the screw pitch).

The following solution of the direct kinematics assumes that the outer axis of the U joints at both upper and lower triangle are directed along the X axis of the triangle (see Fig. 7). Also, it is assumed that the helical joint (lead screw's nut) is placed at the upper triangle and a prismatic joint at the lower.

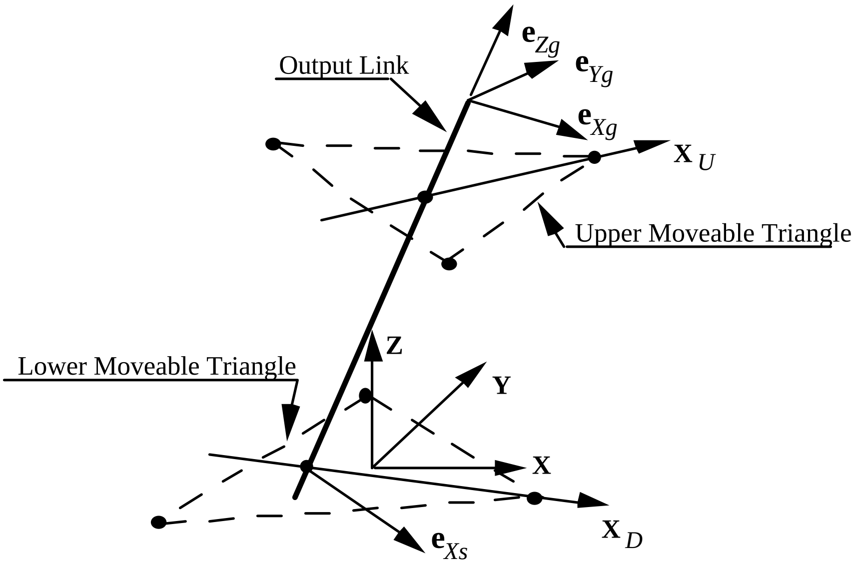

Fig. 7: Skeleton of the six double circular-triangle parallel manipulator

The screw axis direction coincides with the Z axis of the end-effector, $\mathbf{e}_{Zg}$, and is given by:

$$\mathbf{e}_{Zg} = \mathbf{r}_s / \left|\mathbf{r}_s\right|, \tag{12}$$

where $\mathbf{r}_s = \left[x_0^u - x_0^d \quad y_0^u - y_0^d \quad H\right]^T$ is the vector from the lower triangle center to the upper one, H is the distance between the two triangles, and superscripts u and d are for the upper and lower triangle, respectively.

The rotation of the end-effector about the line connecting the two centers is determined by the rotation of the lower triangle which is connected through a U and prismatic joint to the output link. Keeping in mind that three axes; $\mathbf{e}_{zg}$, $\mathbf{e}_{xg}$, and $\mathbf{e}_{XD}$ remain in one plane throughout the motion, the following relation exists:

$$\mathbf{e}_{Zg} \times \mathbf{e}_{Xg} = \frac{\mathbf{e}_{Zg} \times \mathbf{e}_{XD}}{\left|\mathbf{e}_{Zg} \times \mathbf{e}_{XD}\right|}, \tag{13}$$

where $\mathbf{e}_{XD}$ is a unit vector along the X-axis of the lower triangle and can be taken parallel to, for example, $\mathbf{e}_1$ which has been solved for in Section 4.1. Solving this equation for the unit vector $\mathbf{e}_{xg}$, the rotation of the end-effector about the line connecting the two centers is obtained.

The same process is used to obtain the rotation of the nut of the helical joint from which the translational motion in derived.

$$\mathbf{e}_{Zn} \times \mathbf{e}_{Xn} = \frac{\mathbf{e}_{Zn} \times \mathbf{e}_{XU}}{\left|\mathbf{e}_{Zn} \times \mathbf{e}_{XU}\right|}. \tag{14}$$

Here too $\mathbf{e}_{XU}$ can be considered parallel to $\mathbf{e}_1$ of the upper triangle and $\mathbf{e}_{Zn}$ and $\mathbf{e}_{Xn}$ are X and Z axes of the lead screw's nut.

The translation of the end-effector is determined by the difference between the lower (prismatic joint) and the upper triangle (nut) rotational motions. The angle, φ, between the X axes of the end-effector and the lead screw's nut is calculated by:

$$\varphi = \tan^{-1}\left[\left(\mathbf{e}_{Xn} \times \mathbf{e}_{Xg}\right) \cdot \mathbf{e}_{Zg} / \mathbf{e}_{Xn} \cdot \mathbf{e}_{Xg}\right] + 2\pi\left(n_u - n_d\right), \qquad (15)$$

where $\tan^{-1}$ uses the atan2 function and n_d *and* n_u are the number of full revolutions of the upper and lower triangles respectively.

Finally, the position of the gripper is obtained by:

$$\mathbf{P} = \mathbf{R}_{Ou} + \left(L_0 + h \cdot \varphi\right)\mathbf{e}_{Zg}, \qquad (16)$$

where $\mathbf{R}_{Ou}$ is the position vector of the center of the upper triangle, L_0 is the initial length of the upper part of the screw and h is a pitch of the screw.

5. Conclusions

The structure of a new type of six-DOF parallel robot is described in this paper. It consists of two similar structures of a stationary circle and a moving triangle. Each set can actuate the moving triangle in a planar three-DOF motion. This idea, which is a modification of the double-triangular planar manipulator, enables a much larger work envelope, especially when rotational motion is required. The unlimited rotational motion of the moveable triangle obtained in this configuration, enables further extension into a six-DOF manipulator. When two such sets are used, a line connecting the centers of the movable triangles can be positioned in four-DOF. The robot's end-effector is attached to a link along this line while rotation about and translation along this line is achieved by a combination of the additional rotational DOF of each movable triangle. The solution of the direct kinematics of this six-DOF manipulator is given in a closed-form and it is shown that at most, four different solutions exist.

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